Exercise 16

Prove the statement using the ε, δ definition of a limit and illustrate with a diagram like Figure 9.

$$\lim_{x\to 4}(2x-5) = 3$$

Solution

According to Definition 2, proving this limit is logically equivalent to proving that

$$\text{if} \quad |x-4| < \delta \qquad \text{then} \qquad |(2x-5)-3| < \varepsilon$$

for all positive ε. Start by working backwards, looking for a number δ that's greater than $|x-4|$.

$$|(2x-5)-3| < \varepsilon$$

$$|2x-8| < \varepsilon$$

$$|2(x-4)| < \varepsilon$$

$$2|x-4| < \varepsilon$$

$$|x-4| < \frac{\varepsilon}{2}$$

Choose $\delta = \varepsilon/2$. Now, assuming that $|x-4| < \delta$,

$$\begin{aligned}
|(2x-5)-3| &= |2x-8| \\
&= |2(x-4)| \\
&= 2|x-4| \\
&< 2\delta \\
&= 2\left(\frac{\varepsilon}{2}\right) \\
&= \varepsilon.
\end{aligned}$$

Therefore, by the precise definition of a limit,

$$\lim_{x\to 4}(2x-5) = 3.$$

Below is an illustration like Figure 9.